Math Virtual Learning

AP Statistics

Power Part 2

April 16th, 2020

Lesson: April 16th, 2020

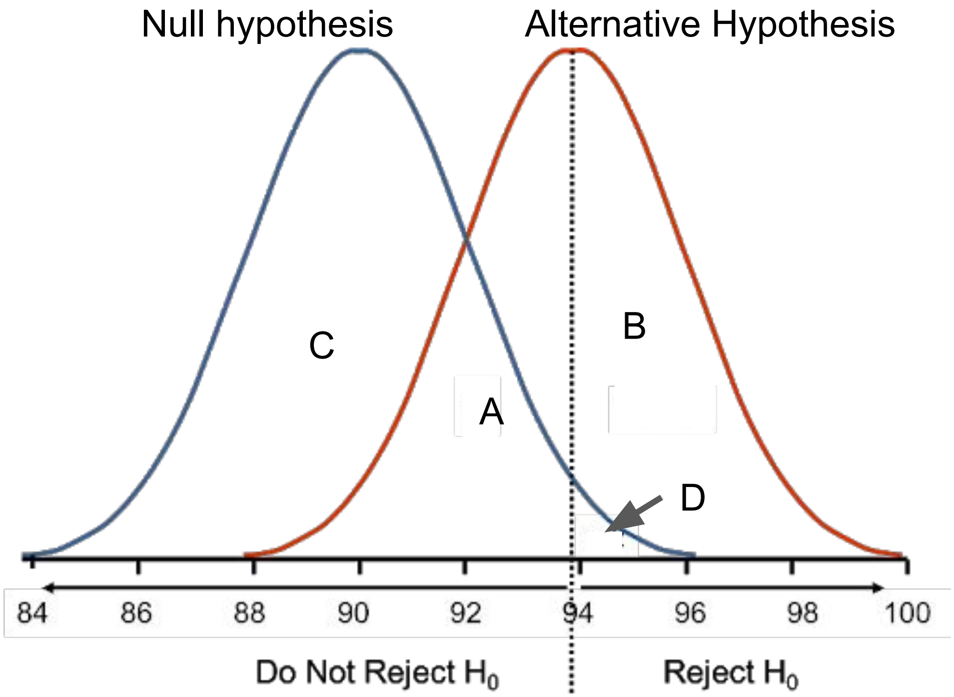
Objective:

Students will extend their knowledge of power, to include the effects of sample size and alternative hypothesis values.

Power Review

Recall, that power is the likelihood that a statistical test is able to detect the difference between the null and true value. The stronger the power, the better your test will be at finding a significant result.

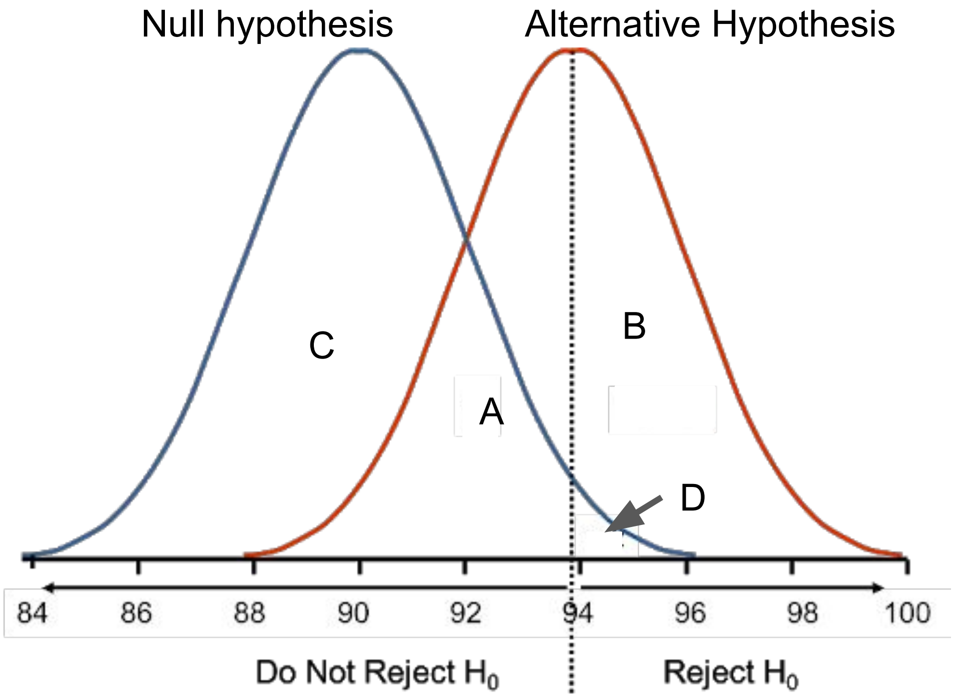
Power Review Question



Examine the drawing to the right. The blue curve represents the sampling distribution if the null is true. The red curve represents the distribution if the Alternative is true.

Where on the diagram would the power of the test be located? Choose the letter representing the area.

Power Review



Area B represents the power of the test. When in this area, the test will reject the null when it should. Areas A (inside the red curve) and D(inside the blue curve) represents different errors. Area C has no specific meaning for our purposes.

Power of a test

Today we are going to cover two parts of understanding the power of a test.

The first is going to cover the changes to power that happen when we change the sample size. If we remember, increasing sample size, shrinks the variation in the sampling distribution.

The second is going to cover the changes to power that happen when the true value of the proportion changes. Thinking what happens to the power when the mean of the red and blue distributions in the previous problem are further apart.

What if they changed the sample size?

In the previous activity, the students randomly selected 65 boxes in performing their test of significance. They tested the following hypotheses,

$$H_O: p = 0.20 \qquad \text{where } p = \text{the proportion of all boxes with the voucher}$$
$$H_A: p < 0.20$$

It was calculated previously, via simulation, that the students' test, using $\alpha = .05$, has a power of approximately 0.226 against the alternative hypothesis of $p = 0.15$.

What would happen to the power against $p = 0.15$ if the sample size was increased? This is the subject of the investigation below.

What if they changed the sample size?

Suppose the students decide to perform a second test, only this time they will randomly select 130 boxes. If the students use the same hypotheses as in their first 65 box test and use $\alpha = .05$, they would have the following rule for concluding the company is cheating.

Conclude the company is cheating if you obtain 18 or fewer boxes with vouchers out of 130.

9. Verify that the rule given above for concluding the company is cheating is correct.

Use the 1-prop z test, and try different numbers of success with n=130 and the appropriate hypothesis. Make sure what they are claiming is true.

What if they changed the sample size?

10. Pretend the company is cheating with $p = 0.15$. Simulate the selection of a random sample of 130 cereal boxes from a population in which 15% of all boxes contain a voucher. How many boxes with vouchers did you obtain? Based on your result, do you have enough evidence to conclude that the company is cheating?
(Hint: randBin(130,.15))

What if they changed the sample size?

11. Repeat your simulation from question #10 twenty times and record your results in the table below.

Trial	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Voucher Boxes																				

What if they changed the sample size?

12. Remember, the assumption in the simulation is the company is cheating—$p = 0.15$. Out of your 20 trials in question #11, in how many of them did you conclude that the company is cheating?

What if they changed the sample size?

13. Comment on which sample size—$n = 65$ or $n = 130$—would result in the higher power against the alternative $p = 0.15$. Why do you think this change has happened?

What if they changed the sample size? - ANSWERS

9. Verify that the rule given above for concluding the company is cheating is correct.

   The *P*-value for 18 boxes out of 130 is approximately 0.04. 19 boxes or more gives a *P*-value greater than 0.05.

10. Pretend the company is cheating with $p = 0.15$. Simulate the selection of a random sample of 130 cereal boxes from a population in which 15% of all boxes contain a voucher. How many boxes with vouchers did you obtain? Based on your result, do you have enough evidence to conclude that the company is cheating?

    I received 21 boxes with vouchers when I did the simulation. Thus, I do not have enough evidence to conclude the company is cheating. If you received 18 or less you should have evidence

What if they changed the sample size? - ANSWERS

11. Repeat your simulation from question #10 twenty times and record your results in the table below. **I entered my results in the table below, but answers will vary.**

Trial	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Voucher Boxes	17	22	14	21	24	22	15	13	20	17	21	20	13	19	20	24	17	13	22	9

12. Remember, the assumption in the simulation is the company is cheating—$p = 0.15$. Out of your 20 trials in question #11, in how many of them did you conclude that the company is cheating?

Out of the 20 trials, 9 of the trials obtained 18 or fewer boxes. Thus, in 45% of my 20 trials I would conclude that the company is cheating. (Students almost always reject in more trials for $n = 130$ than for $n = 65$.

What if they changed the sample size? - ANSWERS

13. Comment on which sample size—$n = 65$ or $n = 130$—would result in the higher power against the alternative $p = 0.15$.

The larger sample size has the higher power against the alternative of $p = 0.15$. This is because the standard deviation of the sampling distribution decreases as n gets larger. So the distributions are narrower, and thus points that vary from the mean are more uncommon.

WHAT IF THE VALUE OF H_A CHANGES?

What if the company is cheating with a value of p different from 0.15? Good question. Suppose the company is cheating with a value of $p = 0.10$—that is, only 10% of all boxes have the vouchers inside. How likely will it be for the students' 65 box test to catch the company?

14. Predict how the power of the 65 box test using $\alpha = .05$ against the alternative $p = 0.10$ will compare to its power against the alternative $p = 0.15$.

WHAT IF THE VALUE OF H_A CHANGES?

15. In order to test your prediction, run 20 trials of the simulation from question #3—be sure to change the probability used from .15 to .10. (randBin(65, 0.10)). Record your results in the table below.

Trial	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Voucher Boxes																				

WHAT IF THE VALUE OF H_A CHANGES?

16. Remember, the assumption in the simulation is the company is cheating—$p = 0.10$. Out of your 20 trials in question #15, in how many of them did you conclude that the company is cheating? (Recall that 7 or fewer boxes with vouchers results in a rejection in the 65 box test and concludes the company is cheating.)

WHAT IF THE VALUE OF H_A CHANGES?

17. Comment on which alternative—$p = 0.10$ or $p = 0.15$—the 65 box test with $\alpha = .05$ has a higher power against and how the concept of power agrees with your intuitive sense about which value of p would be easier to detect.

WHAT IF THE VALUE OF H_A CHANGES? Answers

14. Predict how the power of the 65 box test using $\alpha = .05$ against the alternative $p = 0.10$ will compare to its power against the alternative $p = 0.15$.

Since an alternative of $p = 0.10$ means that the company is cheating at a more egregious rate, it should be easier to detect their cheating with the 65 box test. This would mean that the power should increase.

15. In order to test your prediction, run 20 trials of the simulation from question #3—be sure to change the probability used from .15 to .10.

I entered my results in the table below. Answers will vary.

Trial	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Voucher Boxes	5	5	8	8	9	7	10	6	9	5	4	6	7	6	4	5	11	4	5	9

WHAT IF THE VALUE OF H_A CHANGES? Answers

16. Remember, the assumption in the simulation is the company is cheating—$p = 0.10$. Out of your 20 trials in question #15, in how many of them did you conclude that the company is cheating? (Recall how many boxes with vouchers the 65 box test must find in order to conclude the company is cheating.)

    Out of the 20 trials, 13 of the trials obtained 7 or fewer boxes. Thus, in 65% of my 20 trials I would conclude that the company is cheating. Answers will vary.

WHAT IF THE VALUE OF H_A CHANGES? Answers

17. Comment on which alternative—$p = 0.10$ or $p = 0.15$—the 65 box test with $\alpha = .05$ has a higher power against and how the concept of power agrees with your intuitive sense about which value of p would be easier to detect.

The 65 box test has a higher power against the alternative of $p = 0.10$. This makes sense because if the company is putting vouchers in only 10% of the boxes, it should be easier to detect this cheating then if they were putting them in 15% of the boxes. If we think about this in terms of a null and alternative distribution. The larger the difference between the two mean p-hat of the two sampling distributions, the less overlap they have, so the power area gets larger!

Statistical Power

In conclusion, we can see that the power is affected by the the sample size and difference between the true and expected values.

This means that when we are designing studies, we must think ahead about how big of a difference is going to have practical meaning, and make sure we have a large enough sample or experimental group in order to detect that difference.

From a logistical standpoint, we also want to make sure we don’t sample too large, as that could be an unnecessary cost and could break our assumptions about independence. It is a balancing act!